

$$[2] F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

ALL ITEMS WORTH 1 POINT
UNLESS OTHERWISE INDICATED

$$= \int_0^1 e^{-st} t^2 e^{-t} dt + \int_1^{\infty} e^{-st} e^{-t} dt \quad \left(\frac{1}{2}\right)$$

$$= \int_0^1 t^2 e^{-(s+1)t} dt + \lim_{N \rightarrow \infty} \int_1^N e^{-(s+1)t} dt$$

$$\left(\frac{1}{2}\right) = \left(-\frac{1}{s+1} t^2 e^{-(s+1)t} - \frac{2}{(s+1)^2} t e^{-(s+1)t} - \frac{2}{(s+1)^3} e^{-(s+1)t} \right) \Big|_0^1$$

$$+ \left(-\frac{1}{s+1} e^{-(s+1)t} \right) \Big|_1^N \quad \leftarrow \text{MUST BE WRITTEN USING PROPER LIMIT NOTATION}$$

$$\left(\frac{1}{2}\right) = -\frac{1}{s+1} e^{-(s+1)} - \frac{2}{(s+1)^2} e^{-(s+1)} - \frac{2}{(s+1)^3} e^{-(s+1)} + \frac{2}{(s+1)^3}$$

$$+ \left(\lim_{N \rightarrow \infty} -\frac{1}{s+1} (e^{-(s+1)N} - e^{-(s+1)}) \right) \quad \leftarrow \text{MUST BE WRITTEN USING PROPER LIMIT NOTATION}$$

$$= -\left(\frac{1}{s+1} + \frac{2}{(s+1)^2} + \frac{2}{(s+1)^3} \right) e^{-(s+1)} + \frac{2}{(s+1)^3} + \frac{1}{s+1} e^{-(s+1)}$$

$$= \frac{2}{(s+1)^3} - \left(\frac{2}{(s+1)^2} + \frac{2}{(s+1)^3} \right) e^{-(s+1)}$$

IF $s+1 > 0$
IE $s > -1$

$$\left(\frac{2}\right) \begin{array}{l} \frac{u}{t^2} \quad \frac{dv}{e^{-(s+1)t}} \\ + \\ 2t \quad -\frac{1}{s+1} e^{-(s+1)t} \\ - \\ 2 \quad \frac{1}{(s+1)^2} e^{-(s+1)t} \\ + \\ 0 \quad -\frac{1}{(s+1)^3} e^{-(s+1)t} \end{array}$$

$$[3] \quad \frac{7-3s}{(s+2)^4} = \frac{\overset{0}{A}}{s+2} + \frac{\overset{0}{B}}{(s+2)^2} + \frac{\overset{-3}{C}}{(s+2)^3} + \frac{\overset{13}{D}}{(s+2)^4} \quad (2)$$

$$\underline{7-3s = A(s+2)^3 + B(s+2)^2 + C(s+2) + D}$$

$$s = -2: \quad 13 = D$$

$$\text{COEF OF } s^3: \quad 0 = A$$

$$\text{COEF OF } s^2: \quad 0 = B \quad (\text{SINCE } A=0)$$

$$\text{COEF OF } s: \quad -3 = C \quad (\text{SINCE } A=B=0)$$

$$\text{SANITY CHECK: } s = -3$$

$$\frac{7+9}{1} = 16$$

$$\frac{-3}{-1} + \frac{13}{1} = 3+13=16$$

$$\mathcal{L}^{-1} \left\{ \frac{-3}{(s+2)^3} + \frac{13}{(s+2)^4} \right\} = -3 \cdot \frac{1}{2!} t^2 e^{-2t} + 13 \cdot \frac{1}{3!} t^3 e^{-2t}$$

$$= \underline{-\frac{3}{2} t^2 e^{-2t} + \frac{13}{6} t^3 e^{-2t}}$$

$$\left(\frac{1}{2} \right)$$

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$$[4] \quad \underline{s^2 Y + 3s + 10} \quad (1\frac{1}{2})$$

$$-4(sY + 3)$$

$$+20Y = \frac{40}{s-4}$$

$$\underline{(s^2 - 4s + 20)Y + 3s - 2} = \frac{40}{s-4}$$

$$Y = \frac{-3s + 2}{(s-2)^2 + 16} + \frac{40}{(s-4)(s-2)^2 + 16}$$



FOR REMAINING WORK
SEE ALTERNATE
SOLUTION IF YOU
ADDED THESE
FRACTIONS FIRST

$$\frac{-3s + 2}{(s-2)^2 + 16} = \frac{D(s-2) + E(4)}{(s-2)^2 + 16}$$

$$\rightarrow -3s + 2 = D(s-2) + E(4)$$

$$s=2: -4 = 4E \rightarrow E = -1$$

$$\text{COEF OF } s: -3 = D$$

$$= \frac{-3(s-2) - (4)}{(s-2)^2 + 16} \quad (1\frac{1}{2})$$

$$\mathcal{L}^{-1} \left\{ \frac{-3(s-2) - (4)}{(s-2)^2 + 16} \right\} = \underline{-3e^{2t} \cos 4t - e^{2t} \sin 4t} \quad (1\frac{1}{2})$$

$$\frac{40}{(s-4)((s-2)^2 + 16)} = \frac{A}{s-4} + \frac{B(s-2) + C(4)}{(s-2)^2 + 16} \quad (1\frac{1}{2})$$

$$\underline{40 = A[(s-2)^2 + 16] + B(s-2)(s-4) + C(4)(s-4)}$$

$$s=4: \underline{40 = A(20) \rightarrow A=2}$$

$$s=2: \underline{40 = 2(16) + C(4)(-2) \rightarrow 40 = 32 - 8C \rightarrow C = -1}$$

$$\text{COEF OF } s^2: \underline{0 = A + B \rightarrow B = -A = -2}$$

SANITY CHECK: $s=3$

$$\frac{40}{-1(7)} = \frac{-40}{17} \quad (1\frac{1}{2}) \quad \frac{2}{-1} + \frac{-2(1) - 4}{17} = \underline{-2 - \frac{6}{17} = \frac{-34 - 6}{17} = \frac{-40}{17}}$$

$$\mathcal{L}^{-1} \left\{ \frac{2}{s-4} + \frac{-2(s-2) - (4)}{(s-2)^2 + 16} \right\} = \underline{2e^{4t} - 2e^{2t} \cos 4t - e^{2t} \sin 4t} \quad (1\frac{1}{2})$$

$$y = -3e^{2t} \cos 4t - e^{2t} \sin 4t + 2e^{4t}$$

$$-2e^{2t} \cos 4t - e^{2t} \sin 4t$$

$$= \underline{-5e^{2t} \cos 4t - 2e^{2t} \sin 4t + 2e^{4t}} \quad (1)$$

(*)
$$Y = \frac{(-3s+2)(s-4)+40}{(s-4)((s-2)^2+16)} = \frac{-3s^2+14s+32}{(s-4)((s-2)^2+16)} = \frac{2}{s-4} + \frac{-5}{(s-2)^2+16} + \frac{-2(4)}{(s-2)^2+16}$$

ALTERNATE SOLUTION

$$\underline{-3s^2+14s+32 = A[(s-2)^2+16] + B(s-2)(s-4) + C(4)(s-4)}$$

$$s=4: -48+56+32 = A(20) \rightarrow 40 = 20A \rightarrow A=2$$

$$s=2: -12+28+32 = 2(16) + C(4)(-2) \rightarrow 48 = 32 - 8C \rightarrow C = -2$$

COEF OF s^2 :

$$\underline{-3 = A+B \rightarrow B = -3-A = -5}$$

SANITY CHECK: $s=3$

$$\frac{-27+42+32}{-1(17)} = -\frac{47}{17}$$

$$\frac{2}{-1} + \frac{-5(1)-2(4)}{17} = -2 - \frac{13}{17} = \frac{-34-13}{17} = -\frac{47}{17}$$

$$\mathcal{L}^{-1} \left\{ \frac{2}{s-4} + \frac{-5(s-2)-2(4)}{(s-2)^2+16} \right\}$$

$$= \underline{2e^{4t} - 5e^{2t} \cos 4t - 2e^{2t} \sin 4t}$$